Work 功

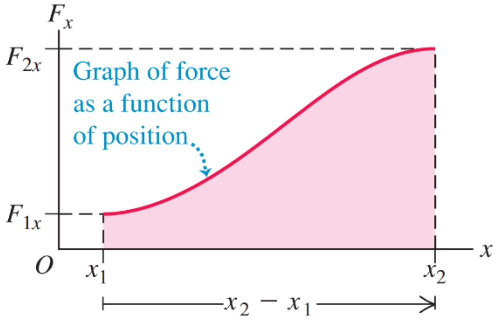
$\vec{F}$ is a constant:

$$W = \vec{F} \cdot \vec{s} = Fs\cos\theta$$

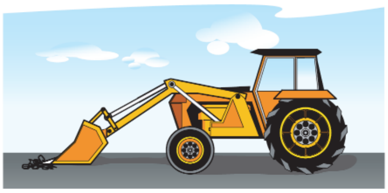
$\vec{F}$ is varying:

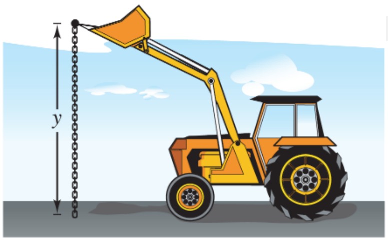
$$W = \lim_{\Delta x \to 0} \sum_{i=1}^{n} F_i \Delta x_i = \int_{x1}^{x2} F_x dx$$

(straight line displacement)



Example: A 20-foot chain weighing 5 pounds per foot is lying coiled on the ground. How much work is required to raise one end of the chain to a height of 20 feet so that it is fully extended, as shown in the right figure.





Imagine that the chain is divided into small sections, each of length dy. Then the weight of each section is the increment of force

$$dF = (weight) = \left(\frac{5\ pounds}{foot}\right) length = 5dy$$

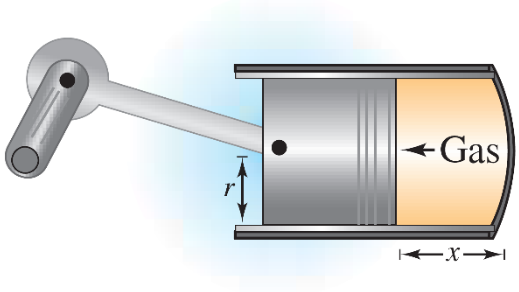
Because a typical section (initially on the ground) is raised to a height of y, the increment of work is

$$dW = (force\ increment)(distance) = dF(y) = 5ydy$$

Because y ranges from 0 to 20, the total work is

$$W = \int_0^{20} 5ydy = \left(\frac{5y^2}{2}\right)_0^{20} = \frac{5(400)}{2} = 1000 \quad (foot \cdot pound)$$

Example: A quantity of gas with an initial volume of 1 cubic foot and a pressure of 500 pounds per square foot expands to a volume of 2 cubic feet. Assume that the pressure is inversely proportional to the volume. Find the work down by the gas.



Pressure-volume: $p = k/V \quad \Rightarrow \quad k = pV = 500 \cdot 1 = 500$

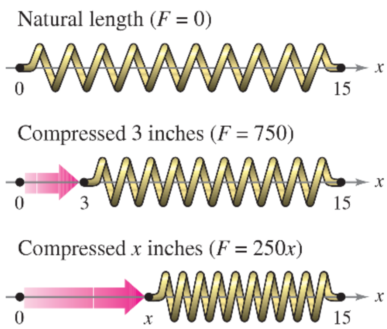
Volume-displacement: $V = sx \Rightarrow dV = sdx$

where s is the area of the cylinder

$$Fdx = psdx = pdV = (k/V)dV$$

$$W = \int_{V0}^{V1} \frac{k}{v} dV = \int_1^2 \frac{500}{v} dV = (500\ln|V|)_1^2 \approx 346.6 \ (foot \cdot pound)$$

Exercise 12: A Force of 750 pounds compresses a spring 3 inches from its natural length of 15 inches. Find the work down in compressing the spring an additional 3 inches.



By Hooke's law:

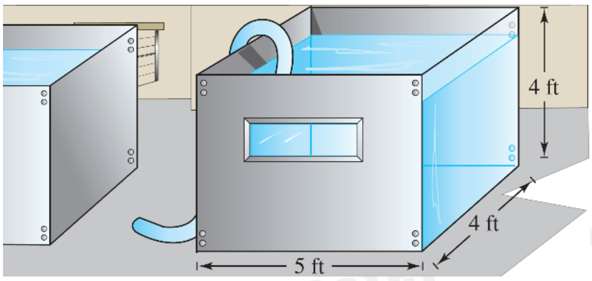
$F(x) = kx \ \Rightarrow$

$k = F/x = 750/3 = 250 \quad (pound/inch)$

The spring will be compressed from 3 inches to 6 inches

$$W = \int_{x1}^{x2} F_x dx = \int_3^6 (250x)dx = (125x^2)_3^6 = 3375\ (pound \cdot inch)$$

Exercise 13: A rectangular tank with a base 4 feet by 5 feet and a height of 4 feet is full of water. The water's density is ρ=62.4 pounds per cubic foot. How much work is done in pumping water out over the top edge in order to empty half of the tank?



Water's mass: $m=\rho V,\ V=sx,\ s = 4 \cdot 5$ (square feet)

$$\text{Work: } \mathrm{W} = \int_0^2 F dx = \int_0^2 mg dx = \int_0^2 \rho V dx = \int_0^2 \rho s x dx = \left(\rho s \frac{x^2}{2}\right)_0^2$$

$$= 62.4 \cdot 4 \cdot 5 \cdot \frac{4}{2} = 2496\ (feet \cdot pound)$$

Work done by gravitational force near the surface of the Earth

For an object of mass m moves from p_1 to p_2 along path L.

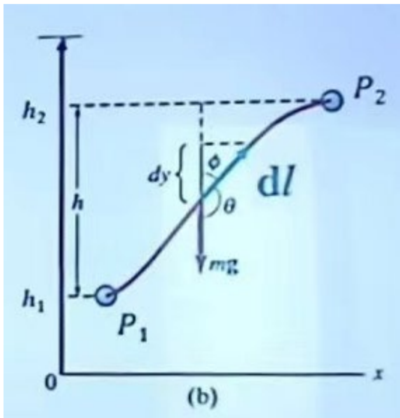
$\vec{F} = -mg\vec{j}$

$d\vec{l} = dx\vec{i} + dy\vec{j} + dz\vec{k}$

$dW = \vec{F} \cdot d\vec{l} = -mgdy$

$$\Rightarrow W = \int \vec{F} \cdot d\vec{l} = \int_{h_1}^{h_2} -mgdy$$

$$= -(mgh_2 - mgh_1)$$



Conservative force and potential energy 保守力和势能

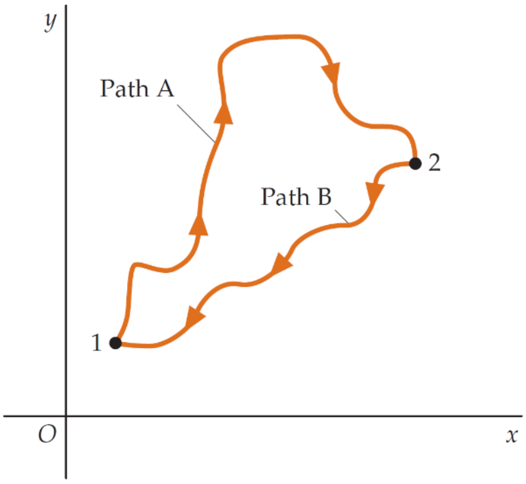
For a conservative force:

$$W = \int_L \vec{F} \cdot d\vec{l} = -\left(E_p(B) - E_p(A)\right)$$

- E_p is a function only depend on the position of the object.
- E_p has the unit of energy.

We denote the function E_p as the potential energy of the corresponding conservative force F.

For each conservative force, there is always a potential energy associated with the force.



Kinetic-Energy and the Work-Energy Theorem 功能定理

Kinetic Energy

$$E_k = \frac{1}{2}mv^2$$

Work-Energy Theorem

$$W_{tot} = E_{k2} - E_{k1} = \Delta E_k$$

Work-Energy Theorem for a Particle System

Considering a system consists of n particles: work done on the i-th particle is W_i

$$W_i = \Delta E_{ki}$$

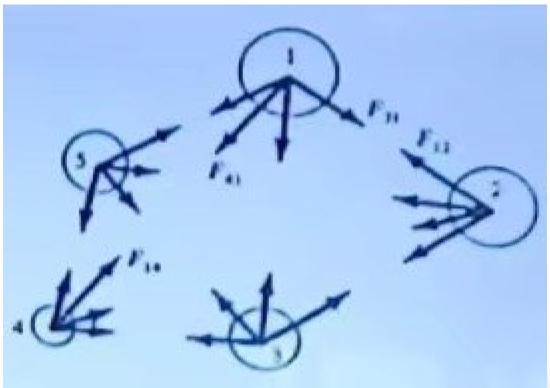
For all of the particles:

$$\sum W_i = \sum \Delta E_{ki}$$

$$\Rightarrow \sum W_i = \boxed{W_{t-int}} + \boxed{W_{t-ext}}$$

Work done by internal force

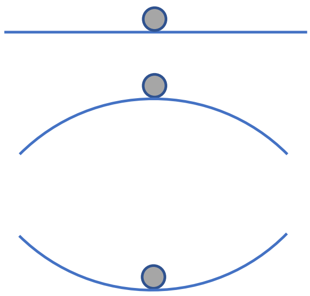
Work done by external force

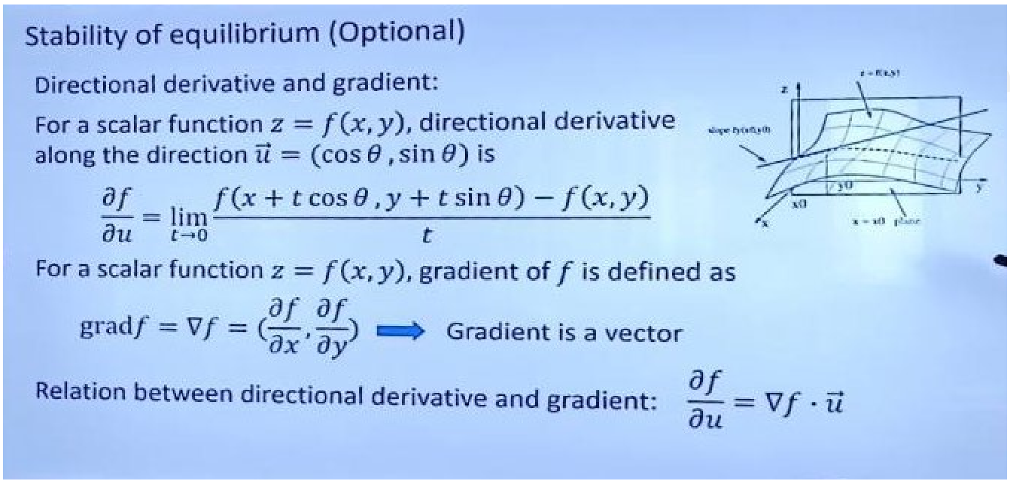


Stability of Equilibrium 平衡的稳定性

Property of equilibrium – stability

Neutral equilibrium : no force arise from a small displacement of the object from equilibrium that tends to drag it either backward, or away from, its original position.





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